Statistics and Bioinformatics
Problem set 8
Due in class, December 7

Practice exercises

1) The t-value is an appropriate test to use when the _____ (true standard deviation, true mean) is not known but must be calculated from the data.

2) The power of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

3) The sensitivity of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

4) The alpha level of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

5) The beta level of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

6) The significance level of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

7) The specificity of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

8) The Type I error of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

9) The Type II error of a test is the probability that the _____ (null, alternative) hypothesis is _____ (retained, rejected) given that the _____ (null, alternative) hypothesis is _____ (true, false).

10) The probability *p* calculated for a test result is the probability of getting a test result that is equally or _____ (more, less) extreme, given that the _____ (null, alternative) hypothesis is _____ (true, false).

11) The power of a test **increases** with increased _____ (sample size, standard deviation, true difference between H_A and H_0, alpha).

12) The power of a test **decreases** with increased _____ (sample size, standard deviation, true difference between H_A and H_0, alpha).

13) If the null hypothesis is false, then the probability of rejecting it **increases** with increasing _____ (sample size, standard deviation, true difference between H_A and H_0, alpha).

14) If the null hypothesis is false, then the probability of rejecting it **decreases** with increasing _____ (sample size, standard deviation, true difference between H_A and H_0, alpha).

15) If the null hypothesis is true, then the probability of rejecting it _____ (increases, decreases, stays the same) with increasing sample size.

16) If the null hypothesis is true, then the probability of rejecting it _____ (increases, decreases, stays the same) with increasing alpha.

17) If the null hypothesis is true, then the probability of rejecting it _____ (increases, decreases, stays the same) with increasing standard deviation.

18) If the alternative hypothesis is true, then power is equal to ___1___ if the sample size is infinite.

19) Using the R function power.examp(), answer the following:

a) For an alpha = 0.05, standard deviation of 1, and true difference of 0.1, what sample size is necessary for a power of 0.5? What sample size is necessary for a power of 0.9? For a power of 0.95? For a power of 0.99? What is the relationship between sample size and power?

$$n = \frac{(z_{pow} - z_{crit})^2 \sigma^2}{\delta^2} = \frac{(z_{pow} - 1.64)^2 (1)^2}{0.1^2}$$

0.5: n = 271 0.95: n = 1082

0.9: n = 856 0.99: n = 1577

Power increases as n increases

b) For a standard deviation of 1, true difference of 0.1, and sample size of 500, what is the power for an alpha of 0.05? for an alpha of 0.01? for an alpha of 0.1? What is the relationship between alpha and power?

$$z_{pow} = z_{crit} - \frac{\delta\sqrt{n}}{\sigma} = 1.64 - \frac{(0.1)\sqrt{500}}{1}$$

$$\text{Power} = 1 - \text{pnorm}(z_{pow})$$

0.05: power = 0.72 0.1: power = 0.83

0.01: power = 0.46

Power increases as alpha increases

Problem

20) A dealer sells a Grade A brand of tomatoes. He advertises a guarantee: he will give customers 100 HRK for every tomato they find on the shelf weighing less than some minimum amount. He monitors the tomatoes from each grower by weighing a random sample from each grower, and asking Is the true mean tomato weight of the grower less than 200 g? If the true mean weight from the grower is greater than or equal to 200 g, he can sell them as Grade A and make more money. If the true mean weight of tomatoes from the grower is less than 200 g, then he must sell them as Grade B. However, he can't weigh all the tomatoes that he sells from each grower, he can weigh only a sample of 100. He wants to calculate the sample mean weight, and perform the appropriate hypothesis test.

a) Write down the null hypothesis and the alternative hypothesis.

$H_A: \mu < 200$ alternative

$H_0: \mu \geq 200$ null

b) Is this a 2-tailed, right-tailed, or left-tailed test?

Left-tailed

c) If the dealer wants his false positive rate to be 0.05, what is the probability in one tail of the distribution?

0.05

d) The false positive rate is the probability that the dealer _____ (accept, reject) the null hypothesis given that the _____ (null hypothesis, alternative hypothesis) is _____ (true, false).

e) Assume the standard deviation in tomato weight is known to be 20 grams. What is the standard deviation (or S.E.M.) of the sample mean weight, for the dealer's sample size?

$$SEM = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2$$

f) Assume that he performs a Z test. Write the equation for the Z value that he will use.

$$Z = \frac{\bar{x} - 200}{2}$$

g) What is the critical Z for his hypothesis test? (use qnorm(), or Tablica A, p. 297 of Vasilj)

$$Z_{crit} = 1.64 = \text{qnorm}(0.95)$$

h) What is the least significant difference (the numerator of the Z) that would reject the null hypothesis, at an alpha probability of 0.05?

$$LSD = Z_{crit} \cdot \frac{\sigma}{\sqrt{n}} = 1.64 \cdot 2 = 3.28$$

i) If the mean tomato weight in a sample of 100 tomatoes is less than 196.72, then the dealer sells that grower's tomatoes as Grade B, assuming alpha = 0.05.

j) Let's say the dealer is willing to accept an alpha probability of 0.10. What is the least significant difference at alpha = 0.10?

$$Z_{crit} = \text{qnorm}(1-\alpha) = \text{qnorm}(0.9) = 1.28$$

$$LSD = Z_{crit} \frac{\sigma}{\sqrt{n}} = 1.28 \cdot 2 = 2.56$$

k) If the mean tomato weight in a sample of 100 tomatoes is less than 197.44, then the dealer sells that grower's tomatoes as Grade B, assuming alpha = 0.10.

l) If the true weight of the grower's tomatoes is 200 g, what is the probability that the dealer will reject the null hypothesis?

alpha (α) by definition

m) If the true weight of the grower's tomatoes is 199 g, what is the probability that the dealer will reject the null hypothesis? What is the power of the hypothesis test?

$z_{pow} = z_{crit} - \frac{\delta\sqrt{n}}{\sigma} = 1.64 - \frac{(1)\sqrt{100}}{20} = 1.64 - 0.5 = 1.14$

power = 1 − pnorm(zpow) = 1 − pnorm(1.14) = 0.13

n) If the true weight of the grower's tomatoes is 198 g, what is the probability that the dealer will reject the null hypothesis? What is the power of the hypothesis test?

$\delta = 2$, $z_{pow} = 1.64 - (2)(0.5) = 1.64 - 1 = 0.64$

power = 1 − pnorm(0.64) = 0.26

o) What does the true weight of the grower's tomatoes have to be in order for the dealer to reject the null hypothesis at a probablity of 50%? 80%? 90%?

z_{pow}

$z_{0.5} = 0$

$z_{0.2} = -0.84$

$z_{0.1} = -1.28$

= qnorm(1 − power)

$\delta = \frac{(\sigma)(z_{crit} - z_{pow})}{\sqrt{n}}$

$\sigma = 20$ $z_{crit} = 1.64$

$\sqrt{n} = 10$

50%: $\delta = 3.28$ $\mu = 196.72$

80%: $\delta = 4.96$ $\mu = 195.04$

90%: $\delta = 5.84$ $\mu = 194.16$

p) List the assumptions that the dealer must for this power analysis to be valid.

Tomatoes must be randomly sampled from the grower, and the sample mean weight should be normally distributed, but it will be normal for a sample size of 100.