Lecture 15: Type I, II errors, power of a hypothesis test

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- Today: statistical power, and errors of a hypothesis test.
- Thursday: review session on all material for the test, especially this week's problem set.

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If you do not reject the null hypothesis, that could be because:
1) the null hypothesis is true, or
2) your test was not powerful enough: your sample size was too low.

What is the power of a test?

Problem

As you recall, a screening test has four combinations of test outcome/condition.

Example

As you recall, a screening test has four combinations of test outcome/condition.

Errors

Situation	Decision	
	Accept H_0	Reject H_0
H_0 true	$1 - \alpha$	α
H_0 false	β	$1 - \beta$

Terminology

- **Type I error** is α.

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- **Type I error** is α.
- **Type II error** is β.
- **Sensitivity** of the test is the power, which is $1 - \beta$.
- **Specificity** of the test is $1 - \alpha$.

Basic power equations, one-sample Z-test

$$Z_{pow} = |Z_{crit}| - \frac{\delta}{\frac{\sigma}{\sqrt{n}}} = |Z_{crit}| - \frac{\delta\sqrt{n}}{\sigma}$$

$$n = \sigma^2 \frac{(Z_{pow} - Z_{crit})^2}{\delta^2}$$

$$\delta = \sigma \frac{(Z_{crit} - Z_{pow})}{\sqrt{n}}$$

Calculating power using R

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$$Z_{pow} = \text{qnorm}(1 - \text{power})$$

$$Z_{crit} = \pm\text{qnorm}(1 - \alpha/T),$$

T is the number of tails in the test. Note that the Z_{crit} is negative for a left-tailed test. + for right - tailed

Example

A buyer wants to buy a large batch of apples from a grower. But he does not want to buy if the true mean weight is less than 150 g. He can't weigh all the apples, but he can weigh a sample of n, and calculate the sample mean. Let's say the true standard deviation is 30g. If he weighs 20 apples and does a Z test he may or may not reject the null hypothesis. If the **true** mean weight of apples is 150 g, what is the probability that he rejects the null hypothesis, and rejects the apples? Assume he is doing a one-tailed Z-test, with a significance level of 0.05 ($\alpha = 0.05$). He rejects the apples only if he rejects the null hypothesis.

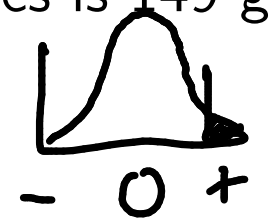
Example

$\sigma = 30 \quad n = 20$

$\delta = 1g \quad \alpha = 0.05$

$z_{crit} = 1.64$

What if the true mean weight of the grower's apples is 149 g? Will he reject the null hypothesis? 145 g? 140 g?



$$z_{pow} = z_{crit} - \frac{\delta\sqrt{n}}{\sigma}$$

$$= 1.64 - \frac{1\sqrt{20}}{30} = 1.49$$

power = 0.068

Example

$$LSD = z_{crit} \frac{\sigma}{\sqrt{n}}$$

What is the least significant difference (LSD) that the buyer must see before he rejects the grower's apples?

$$LSD = \underline{1.64} \cdot \frac{30}{\sqrt{20}} = 11.0$$

Decision: reject apples if $\bar{x} < 139$
if $n = 20$, $z_{crit} = 1.64$, $\sigma = 30$

Example

What is the probability that he will see the LSD if the true mean weight of the grower's apples is 149 g? 145 g? 140 g?

Example

$z_{crit} = 1.64$ $n = 20$ $\sigma = 30$ power $= 0.8$

$$\delta = \sigma \frac{(z_{crit} - z_{pow})}{\sqrt{n}}$$

z_{pow} = qnorm(1 − pow) = −0.84

Statistical power

If his sample size has to be 20, then what true difference (δ) will he be able to detect with his test 80% of the time? What if the sample size is 30? 50? 100? 500?

$$\delta = 30 \frac{(1.64 - (-0.84))}{\sqrt{20}} = 16.6$$

Example

$\sigma = 30$ $z_{pow} = -0.84$ $z_{crit} = 1.64$

$$n = \frac{\sigma^2 (z_{pow} - z_{crit})^2}{\delta^2}$$

$\delta = 2$

If he wants to detect, at a power of 80%, a true difference of only 2 g, what sample size will he need? 1 g? 0.5 g?

Example

If he wants to detect a true difference of 10 g, at a power of 80%, what should his sample size be? At a power of 90%? 95%? 99%?

Example

If he wants to detect a true difference of 15 g, at a power of 80%, with a sample size of 20. What effect α should he use?