Lecture 14: Hypothesis testing, continued

Announcements:

- Reading for this week's subject: pp. 61 - 79 in Vasilj.

Hypothesis testing: example

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Hypothesis testing: example

We have a new variety of tomato. Does it have **the same** resistance to fusarium wilt as the old variety? To answer this question, we do an experiment. We expose 100 new plants to fusarium, and found that the mean growth rate was 35 with standard deviation 10. We know from previous experience that the mean growth rate of the old genotype is 33.

Hypothesis testing: procedure

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8. If the statistic is in the rejection region, we **reject** the null hypothesis and **accept** the alternative hypothesis.
9. If the statistic is not in the rejection region, we **retain** the null hypothesis, and do not accept the alternative hypothesis.

Rules

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- If the alternative hypothesis contains the $<$ sign, the test is left-tailed.
- If the alternative hypothesis contains the $>$ sign, the test is right-tailed.

Hypothesis testing: example

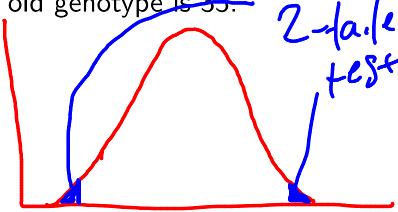
μ_n = resistance of new

μ_o = " " old

We have a new variety of tomato. Does it have **the same** resistance to fusarium wilt as the old variety? To answer this question, we do an experiment. We expose 100 new plants to fusarium, and found that the mean growth rate was 35 with standard deviation 10. We know from previous experience that the mean growth rate of the old genotype is 33.

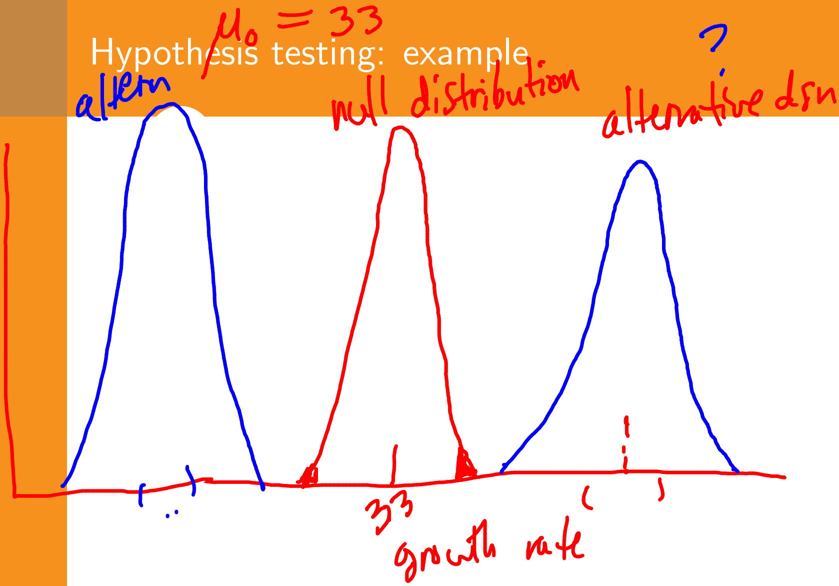
$H_0: \mu_n - \mu_o = 0$

$H_A: \mu_n - \mu_o \neq 0$



2-tailed test

Hypothesis testing: example



Hypothesis testing: example

μ_n new

μ_o old

We have a new variety of tomato. Does it have **lower** resistance to fusarium wilt than the old variety? To answer this question, we do an experiment. We expose 100 new plants to fusarium, and found that the mean growth rate was 35 with standard deviation 10. We know from previous experience that the mean growth rate of the old genotype is 33.

Left-tailed test

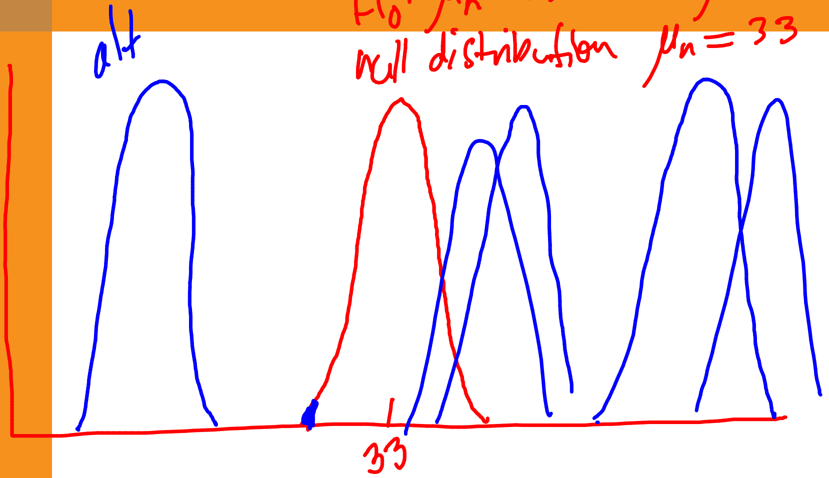
$H_A: \mu_n - \mu_o < 0$

$H_0: \mu_n - \mu_o \geq 0$

Hypothesis testing: example

$H_0: \mu_n - 33 \geq 0 \Rightarrow \mu_n \geq 33$

null distribution $\mu_n = 33$



Hypothesis testing: example

We have a new variety of tomato. Does it have **higher** resistance to fusarium wilt than the old variety? To answer this question, we do an experiment. We expose 100 new plants to fusarium, and found that the mean growth rate was 35 with standard deviation 10. We know from previous experience that the mean growth rate of the old genotype is 33.

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1 - pnorm(z) = P

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- In a two-tailed test, the probability is twice the tail probability. In a one-tailed test, the probability is the one-tail probability.
- This probability is also called the **significance** or the **significance level** of the test.

Some more terminology

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Hypothesis testing: example

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}} = \sqrt{\frac{S_x^2}{n}}$$

We have a new variety of tomato. Does it have **the same** resistance to fusarium wilt as the old variety? To answer this question, we do an experiment. We expose 100 new plants to fusarium, and found that the mean growth rate was 35 with standard deviation 10. We know from previous experience that the mean growth rate of the old genotype is 33.

$$t = \frac{\bar{x} - \mu_0}{S_{\bar{x}}} = \frac{35-33}{\frac{10}{10}} = 2.$$

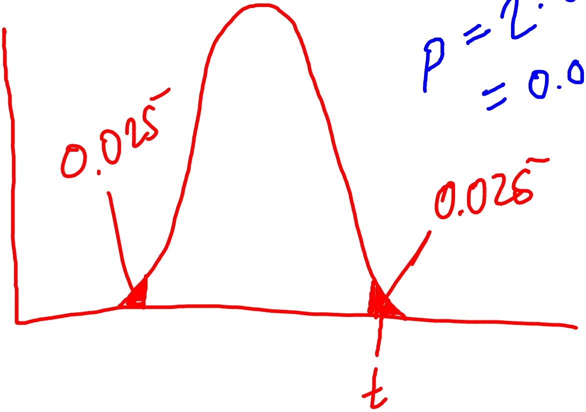
Hypothesis testing: example

$p = P(\text{our test result} \mid H_0 \text{ is true})$

$t = 2 > 1.98 - t_{crit}$

$\therefore$ reject H_0

$p = 2 \cdot 0.0241 = 0.0482$



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Hypothesis testing: example

Unequal variance t-test

Calculate separate variances for the two samples:

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Hypothesis testing: example

Hypothesis testing: one-sample test of proportion example

We have a new variety of tomato. Does it have **the same** resistance to fusarium wilt as the old variety? To answer this question, we do an experiment. We expose 100 new plants to fusarium, and found that 72 are resistant. We know in advance that 65% of the old plants are resistant.

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Hypothesis testing: two-sample test of proportions example

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Hypothesis testing: example

Two sample test of proportions

Given x_1 successes in a sample of n_1, and x_2 successes in a sample of n_3. Therefore $\hat{p}_1 = x_1/n_1$, and $\hat{p}_2 = x_2/n_2$.
We define $\overline{p} = \frac{x_1+x_2}{n_1+n_2}$, and $\overline{q} = 1 - \overline{p}$.
Then our test statistic is:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}}$$