Lecture 11: Z tests and t tests, continued

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- Also, Problem sets 4 through 6 are all due IN CLASS ON TUESDAY, NOVEMBER 23.

More announcements

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- If you have any questions about the exams, please see me or Tomislav.
- Next exam will include all questions in this exam that less than 50% of people answered correctly. These are questions 9, 24, 25, 21, 16, and 5. So make sure you understand those questions!

Z test in general

$$Z =$$

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If observed statistic is the difference between two sample means, its standard deviation is $\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$.

$= \sqrt{Var(\bar{x}_1) + Var(\bar{x}_2)}$

Example: poll results Z test

$$Z = \frac{\text{obs} - \text{exp}}{\text{sd obs}} = \frac{475 - 510}{\sqrt{npq}}$$

$$\sqrt{npq} = \sqrt{(1000)(0.51)(0.49)}$$

Let's say a poll of 1000 random Croatians found that 47.5% Were in favor of entering the EU. Is this unusual if the true proportion was 51.0% in favor?

$$Z = \frac{-35}{15.8} = -2.21$$

∴ unusual, not in favor

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Example: poll results chi square test

χ^2 instead of z

$$\chi^2 = \sum_i \frac{(obs_i - exp_i)^2}{exp_i} = \frac{(475-510)^2}{510} + \frac{(525-490)^2}{490} = 35^2/510 + 35^2/490$$

4.901

Let's say a poll of 1000 random Croatians found that 47.5% Were in favor of entering the EU. Is this unusual if the true proportion was 51.0% in favor?

Observed: 475 favor, 525 against

Expected: 510 favor, 490 against

Example: poll results chi square test

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Chi square test

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Chi square test

$$\chi^2 = \frac{\sum_1^n(\text{count of observed} - \text{count of expected})^2}{\text{count of expected}}$$

Degrees of freedom is the number of counts that are free to vary in this calculation.

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Z test versus t test

$$Z = \frac{\text{obs} - \text{exp}}{\sigma_{\text{obs}}}$$

What if we don't know σ?

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$$t = \frac{\text{observed} - \text{expected}}{s_{\text{obs}}}.$$

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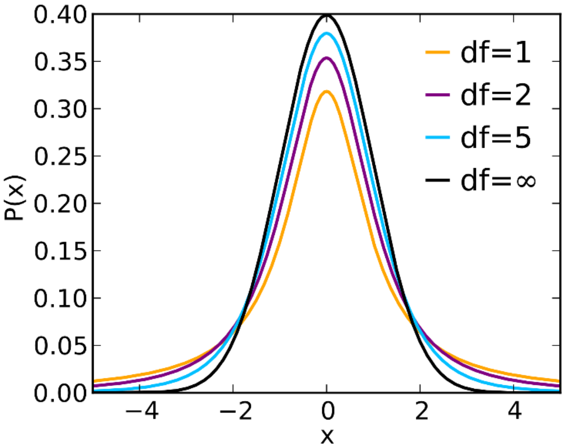
t is very close to normally distributed, symmetrical, with mean of 0 and standard deviation slightly higher than 1.
But t is practically indistinguishable from a normal distribution if $n > 30$.

t distribution for various sample sizes (or degrees of freedom)



Example: a sample of heights

$$t = \frac{\text{obs} - \text{exp}}{\text{sd obs}} = \frac{173 - 169}{9.2/\sqrt{15}}$$

Say we have a random sample of 15 women, their sample mean height is $\overline{x} = 173$ cm, and sample standard deviation of height is $s = 9.2$ cm. Is this unusual if the population mean is $\mu = 169$ cm?

$$= \frac{4}{2.38} = 1.683$$

Example: a sample of heights

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Example: exam scores

In our Exam 1, the mean score of women was 72.7 and the mean score for men was 70.3. The standard deviation for women was 7.7 and for men was 8.3. There were 14 women and 13 men. Is the sex difference in means unusual, if we expect the difference to be zero?

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Degrees of freedom if standard deviations or sample sizes are different in the two groups

Behrens-Fisher problem:
What is the degrees of freedom if $n_1 \neq n_2$ or $s_1 \neq s_2$?

$$\text{d.f.} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}.$$