Lecture 20: Covariance, correlation, and linear regression

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- Reading: Chapter 7 in Vasilj.

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- Physiological relationships.
- Ecological relationships.

Correlation versus causation

Storks

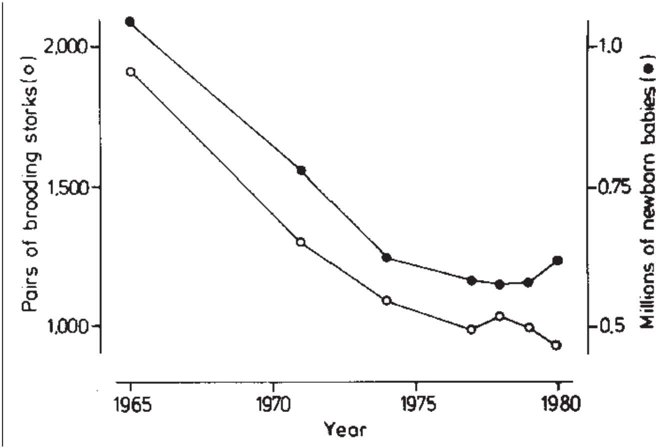
Correlation versus causation

Babies 

Correlation versus causation

Storks and babies?

Correlation versus causation



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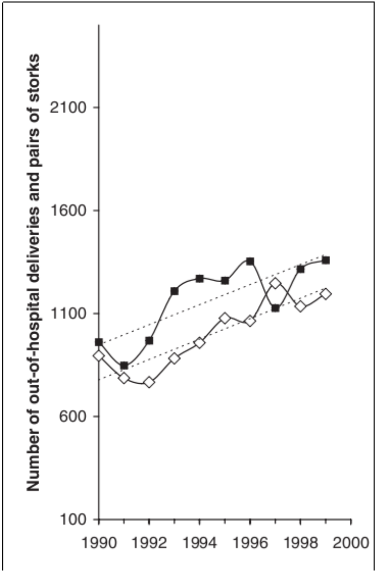

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2. $$Cov(x,y) = \frac{\sum_i [(x_i-\overline{x})(y_i-\overline{y})]}{n-1}$$

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7. If x and y are independent, then $\mathrm{Cov}(x, y) = 0$.

Calculating the slope and intercept of the best-fit line

The best fit line is the line that minimizes the residual sum of squares.

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2. $$\mathrm{Cor}(x, y) = r = \frac{\mathrm{Cov}(x, y)}{s_x s_y}.$$
3. The correlation coefficient lies between -1 and $+1$.

Strength of the correlation coefficient

korelacijski koeficijent (r)	jačina korelacije
0.00 – 0.10	nema
0.10 – 0.25	vrlo slaba
0.25 – 0.40	slaba
0.40 – 0.50	srednja
0.50 – 0.75	jaka
0.75 – 0.90	vrlo jaka
0.90 – 1.00	potpuna

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6. Or let R do it all with anova(lm(y ~ x)).

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3. The residuals are normally distributed
4. The variance is constant (regardless of the value of x)

Interpretation of correlation